Does the non–minimal coupling of the scalar field improve or destroy inflation ?

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Abstract

The non–minimal coupling of a scalar field to the Ricci curvature in a curved spacetime is unavoidable according to several authors. The coupling constant ξ is not a free parameter: the prescriptions for the coupling constant ξ in specific scalar field and gravity theories (in particular in general relativity) are studied. The results are applied to the most popular inflationary scenarios of cosmology and their theoretical consistence is analysed. Certain observational constraints on ξ are also discussed.

1 Non–minimal coupling of the scalar field

The generalization of the flat space Klein–Gordon equation to a curved space,

$$\Box \phi - \xi R \phi - \frac{dV}{d\phi} = 0 \; , \tag{1}$$

includes the possibility of an explicit coupling between the scalar field ϕ and the Ricci curvature of spacetime R. A non–minimal coupling has been advocated and is unavoidable in a quantum theory of ϕ: it is generated by quantum corrections even if it is absent in the classical action, or it is required in order to renormalize the theory. This leads us to ask whether physics selects a unique value (or a range of values) for the coupling constant ξ. What is the value of the coupling constant ξ ? This question is relevant for different areas of theoretical physics, and it is crucial for the application to cosmological inflation (see Ref. [1] and references therein for an overview), the success of which is deeply affected by the value of ξ. The answer to the question depends on the underlying theory of gravity and of the scalar field. A relativist's answer is:

if gravity is described by a metric theory and the scalar field ϕ has a non–gravitational origin[1], and satisfies Eq. (1), then $\xi = 1/6$ ("conformal coupling").

This result, proved in Ref. [2] and later confirmed in Refs. [3], was derived during the study of wave propagation and tails of radiation in curved spaces, and it arises by imposing the Einstein Equivalence Principle (EEP [4]) on the physics of the field ϕ (the structure of tails of radiation becomes closer and closer to that occurring in flat spacetime when the curved manifold is progressively approximated by its tangent space). The result is completely independent of conformal transformations, the conformal structure of spacetime, the spacetime metric g_{ab} and the field equations for g_{ab}; it is, however, unclear why the value of ξ that emerges from this analysis is precisely the one that gives conformal coupling. A naive explanation that can be given is the following: no preferred length or mass scale is present in the flat space Klein–Gordon equation, and therefore no such scale must appear in the corresponding curved space equation for the massless field when small regions of spacetime are considered, if the EEP holds.

The EEP holds in all metric theories of gravity, and it must be imposed on ϕ if ϕ is a

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[1]An example of a scalar field with gravitational origin is the Brans–Dicke field, or its generalization in scalar–tensor theories.

non–gravitational field[2]. If $\xi \neq 1/6$ there is, in principle, the possibility that a *massive* scalar field propagate along the light cones in a space in which $R \neq 0$.

A particle physicist's answer to the problem of the value of ξ is different and more varied:

- if ϕ is a Goldstone boson in a theory with spontaneously broken global symmetry, $\xi = 0$ [5];

- in the large N approximation to the Nambu–Jona–Lasinio model, $\xi = 1/6$ [6];

- in Einstein's gravity with backreaction and $V = \lambda\phi^3$, $\xi = 0$ [7];

- if ϕ is the Higgs field of the standard model $\xi \leq 0$ or $\xi \geq 1/6$ [7];

- in any theory formulated in the Einstein conformal frame $\xi = 0$ (this includes supergravity, the low energy limit of superstring theories, Kaluza–Klein, and virtually all theories involving a dimensional reduction and compactification of the extra dimensions [8]).

Moreover, ξ is subject to renormalization, apparently leaving little room for an unambiguous determination of its value. Fortunately, in cosmology the prospects for the determination of the value of ξ are better than it appears in the general case.

2　Applications to cosmological inflation

The simplification occurs because inflation is a low energy, classical phenomenon: the tensor contributions to the quadrupole in the cosmic microwave background imply that the energy density of the inflaton 60 e–folds before the end of inflation satisfies

$$V_{60} \leq 6 \cdot 10^{-11} m_{pl}^4 \,,$$

where m_{pl} is the Plank mass. Hence, gravity was classical during inflation. The inflaton field is decomposed into its unperturbed value plus quantum fluctuations (that seed density perturbations giving rise to galaxies, clusters and superclusters later in the history of the universe):

$$\phi = \phi_0(t) + \delta\phi(t, \underline{x}) \,. \tag{2}$$

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[2]If ϕ has gravitational origin, statements about its physics pertain to the Strong Equivalence Principle, which is believed to be satisfied only in general relativity [4].

The distribution of ϕ_0 is peaked around classical trajectories and the evolution of ϕ_0 is described by classical equations (see e.g. Refs. [9]).

Cosmologists seem to have two different approaches to the problem of the value of ξ: most authors ignore the problem altogether by setting $\xi = 0$ arbitrarily. Other authors use ξ as a free parameter that can be tuned *a posteriori* to minimize the troubles of specific inflationary scenarios. Instead, the value of the coupling constant is fixed *a priori* in many cases. We analysed the proposed inflationary scenarios by answering the questions:

- is any prescription for ξ applicable ?

- what are the consequences of this prescription for the viability of the scenario ?

The aspects taken into account are the existence of inflationary solutions, a sufficient amount of inflation to solve the problems of the standard big bang model, the fine–tuning of initial conditions, and the evolution of density perturbations. The results of Ref. [1] are summarized in the following (partial) lists, in which "theoretical consistence" refers only to the value of ξ employed in the specific gravity and inflaton theory used – independent arguments may rule out the scenario.

Theoretically consistent scenarios

- Power–law inflation ("PLI") in any theory formulated in the Einstein frame

- Extended inflation (original formulation and recast as PLI)

- Hyperextended inflation (original formulation and recast as PLI)

- Induced gravity inflation

- Natural inflation

- Double field inflation (original formulation and version of Ref. [10])

Theoretically inconsistent scenarios

- New inflation

- Chaotic inflation (general relativity and $V = \lambda \phi^4$)

- Chaotic inflation (general relativity and $V = \mu^2 \left[\phi^2/2 + \lambda/(2n)\phi^{2n} \right]$)

- Chaotic inflation (general relativity and $V = \lambda \left(\phi^2 - v^2 \right)^2$)

- Double field inflation (version of Ref. [11]).

3   Observational constraints on ξ

Adopting a different point of view, it is desirable to constrain the value of the coupling constant ξ by using the available observations of the cosmic microwave background. Unfortunately, despite the fact that some inflationary solutions are known for $\xi \neq 0$, very few predictions have been made for the observables quantitites, in particular for the spectral index n_s of density perturbations. For chaotic inflation with the potential $V = \lambda \phi^4$, one has [12]

$$n_s = 1 - \frac{32\xi}{1 + 960\xi} \ .$$

(3)

The combined statistical analysis of the *COBE* and Tenerife observations yields the 1σ limits $0.9 \leq n_s \leq 1.6$, which imply

$$\xi \leq -1.56 \cdot 10^{-3} \ , \qquad \xi \geq -9.87 \cdot 10^{-4} \ .$$

(4)

The value predicted by general relativity is $n_s = 0.967$. For chaotic inflation with the potential $V = \lambda \left(\phi^2 - v^2 \right)^2$, n_s depends on ξ only to second order in the relevant parameters:

$$n_s = 1 - \frac{2}{60 + \pi \left(v/m_{pl} \right)^2} \ .$$

(5)

The scalar field potential

$$V(\phi) = \lambda \phi^n \quad , \quad n > 6$$

(6)

with $\xi \neq 0$ gives power–law inflation in the following regions of the (n, ξ) parameter space:

$$n > 6 \quad , \quad 0 < \xi < \frac{2}{n^2 - 12n + 44} \ ,$$

(7)

$$6 < n < 4 + 2\sqrt{3} \simeq 7.464 \quad , \quad \xi < 0 \ ,$$

(8)

$$n = 4 + 2\sqrt{3} \quad , \quad \xi < \frac{1}{4(3 - \sqrt{3})} \simeq 0.197 \ ,$$

(9)

$$n > 4 + 2\sqrt{3} \quad , \quad \frac{-2}{n^2 - 8n + 4} < \xi < 0 \ .$$

(10)

The range of values $6 \leq n \leq 10$ is interesting for superstring theories; only a very narrow range of values of ξ is allowed for high n.

4 Conclusions

In most inflationary scenarios, the coupling constant ξ is not a free parameter that can be tuned arbitrarily, but its value is fixed by the theory of gravity and of the scalar field adopted. Moreover, the theoretical consistence of many inflationary scenarios is deeply affected by the value of ξ. Some scenarios turn out to be theoretically inconsistent, while others are viable according to the correct use of non–minimal coupling. The feeling from our and from previous works ([1] and references therein) is that, in general, non–minimal coupling makes it harder to achieve inflation.

Work in progress includes studying the consequences of the prescriptions of ξ for the cosmic no–hair theorems, calculations of density perturbations spectra for $\xi \neq 0$, and the consequence of the general relativity–as–an–attractor behaviour during inflation. Non–cosmological applications of the theory described here include the physics of boson stars and of classical/quantum wormholes.

Acknowledgments

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